

# EMBEDDED DEFECTS AND SYMMETRY BREAKING IN FLIPPED $SU(5)$

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## Abstract

We explicitly show the analogy between the symmetry breaking scheme for the GUT flipped  $SU(5)$  with that of the Weinberg-Salam theory of electroweak interactions. This allows us to construct the embedded defect spectrum of the theory flipped  $SU(5)$ . We find that the spectrum consists of twelve gauge-equivalent unstable Lepto-quark strings, which are analogous to W-strings in electroweak theory, and another string that is gauge inequivalent to the Lepto-quark strings, which we call the ‘V-string’. The V-string is analogous to the Z-string of electroweak theory, correspondingly admitting a stable semilocal limit. Using data on the running coupling constants we indicate that in the non-supersymmetric case V-strings can be stable for part of the physically-viable parameter space. Cosmological consequences are briefly discussed.

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# 1 Introduction.

Flipped  $SU(5)$  [1] is a very special Grand Unified Theory (GUT). It stands out for its ease and simplicity of structure. Furthermore, many of the problems associated with unification are simply not present for this model. One of the practical problems of GUT's is their testability; being physical theories they have to be testable and thus falsifiable. However, GUT's are at such an extreme energy scale that there are few direct tests. Hard predictions, such as proton decay, are few and far between. Additionally, the generality of such predictions tends not to discriminate between rival GUT's. Thus, one must start using arguments such as naturalness and simplicity to focus attention onto your favourite theory. These arguments are not physics, but they do serve to motivate attention onto specific models where, perhaps with more study, they may yield some interesting Cosmology.

For clarity of structure flipped  $SU(5)$  stands proud. It is a very simple theory and its close similarity in structure to the Weinberg-Salam model of Electroweak interactions is a very notable feature (we shall discuss this point later). It has a tiny Higgs structure — requiring only a 10-representation to facilitate symmetry breaking to the Standard Model. Furthermore, flipped  $SU(5)$  is obtainable from fundamental string theory [2].

Problems in physics often indicate new and exciting structure. Generally, the number of such problems increase until a crisis state is reached. The Standard model has a couple of problems in the abstraction of its structure, but by no means is in a critical state of affairs. The standard model's problems notably include the monopole and coupling unification problem [3]. Flipped  $SU(5)$  conveniently solves these problems; stable monopoles are not present [4] and the coupling constants are not required to meet. Also, flipped  $SU(5)$  has a natural see-saw mechanism to guarantee light right-handed neutrinos [5].

Owing to the impossibility of directly testing GUT's in a terrestrial experiment, one turns to the only situation where they were directly relevant — the Early Universe. Some GUT's yield strong, specific Cosmological predictions. Cosmic strings being *the* example [6]. However, there are no topological defects in flipped  $SU(5)$ , rendering it seemingly untestable in this area. Fortunately this happens not to be the case — it does yield non-topological defects, which may be stable. Recent work

on Z-strings in Electroweak theory [7] and the close formal similarity of Electroweak theory to flipped  $SU(5)$  suggests that similar structures may be present for flipped  $SU(5)$ . This work shows that there *are* embedded defects in flipped  $SU(5)$ . Moreover, the counterpart of the Z-string in flipped  $SU(5)$  is very likely stable. Thus flipped  $SU(5)$  should yield a definite (and quite distinct) Cosmological signature. We shall briefly indicate some Cosmological consequences in this paper. More detailed calculations are presently underway.

In section 2 we briefly review the structure of flipped  $SU(5)$ , in such a way as to set the scene for a detailed analysis of symmetry breaking. Symmetry breaking is considered in section 3. Analysis of the symmetry breaking facilitates a discussion of the embedded defects structure, which is discussed in section 4. We find unstable Lepto-quark strings and another string that may be stable, this is called the V-string. Finally in section 5 we discuss the stability and Cosmological consequences of this V-string and summarise our conclusions.

## 2 Flipped $SU(5)$ : The Model.

In this section we express previous work in flipped  $SU(5)$  in a way that will allow a detailed analysis of symmetry breaking and will thus allow us to determine the embedded defect structure of the model. For references see [1], [4], [5] and [8].

Flipped  $SU(5)$  is a Yang-Mills gauge theory with a symmetry breaking potential. The grand unified gauge group is  $SU(5) \times \widetilde{U(1)}$ . In the general case the coupling constants of the simple groups,  $g$  and  $\tilde{g}$  respectively, may be different — as observed from the running couplings in non-supersymmetric gauge theories. Generally, if the couplings do not meet at Grand Unification then a non-simple gauge group is required; such gauge groups are often inspired from Fundamental string theory.

In the following sections, we adopt a notation where fields associated with the  $\widetilde{U(1)}$  part of the gauge group will be denoted with a tilde. This is to make it plain which part of the group is being dealt with.

A modified Gell-Mann basis is used for  $SU(5)$ , as given in the appendix. This basis is orthonormal with respect to the inner product. The Lie algebra for the  $\widetilde{U(1)}$  part is just a phase and is proportional to the identity.

Hence, under its inner product, the Lie algebra decomposes into the direct sum  $L(SU(5)) \oplus L(\widetilde{U(1)})$ . Then the gauge field is denoted by  $A^\mu = A_a^\mu T_a + \widetilde{A}^\mu \widetilde{T}$ , where the sum in  $a$  runs from 1 to 24.

The matter fields transform as the representation  $(n, q_n)$ . Here  $n$  specifies the dimension of the representation of  $SU(5)$  and  $q_n$  is the  $\widetilde{U(1)}$ -charge, which is defined by

$$d_n(\widetilde{T})(n, q_n) = i\sqrt{\frac{12}{5}}q_n(n, q_n), \quad (1)$$

with  $d_n$  the derived  $n$ -dimensional representation of the Lie algebra. Note the normalisation factor of  $\sqrt{12/5}$  coming from the definition of the  $\widetilde{U(1)}$ -generator. Furthermore, an anomaly cancellation [4] yields  $q_n$  as  $q_n = 5 - 2m$ , where  $m$  is the number of anti-symmetric indices labelling the components of the  $n$ -representation.

The fermions are assigned to the following representations of  $SU(5)$ : the trivial 1-representation, the fundamental 5-representation and the antisymmetric 10-representation. These transform under  $SU(5)$  as, respectively:

$$\begin{aligned} D_1(g)M_1 &= M_1, \text{ for } M_1 \in (1, 5), \\ D_5(g)M_5 &= gM_5, \text{ for } M_5 \in (5, 3), \\ D_{10}(g)M_{10} &= gM_{10}g^T, \text{ for } M_{10} \in (10, 1). \end{aligned} \quad (2)$$

The conjugate representations have an  $\widetilde{U(1)}$ -charge of opposite sign.

Fermions are assigned to these representations flipped— $u \leftrightarrow d$ —relative to the usual  $SU(5)$  fermion assignments, so for the left-handed fermions

$$f_L^{10} = (10, 1) = \begin{pmatrix} 0 & \bar{d}_L^1 & -\bar{d}_L^2 & \vdots & d_L^1 & u_L^1 \\ -\bar{d}_L^1 & 0 & \bar{d}_L^3 & \vdots & d_L^2 & u_L^2 \\ \bar{d}_L^2 & -\bar{d}_L^3 & 0 & \vdots & d_L^3 & u_L^3 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ -d_L^1 & -d_L^2 & -d_L^3 & \vdots & 0 & \bar{\nu}_L \\ -u_L^1 & -u_L^2 & -u_L^3 & \vdots & -\bar{\nu}_L & 0 \end{pmatrix}, \quad (3)$$

$$\bar{\mathbf{f}}_L^5 = (\bar{\mathbf{5}}, -3) = \begin{pmatrix} \bar{u}_L^1 \\ \bar{u}_L^2 \\ \bar{u}_L^3 \\ \dots \\ \nu_L \\ \bar{e}_L^+ \end{pmatrix}, \quad (4)$$

$$\mathbf{f}_L^1 = (1, 5) = e_L^-. \quad (5)$$

The right-handed fermions are assigned to the conjugate representations. Fermions have been so assigned to have the correct observed hypercharges (see eq. (20)). The flipped representations are tied in to the non-trivial symmetry breaking, which is further tied in to the Higgs representation chosen for symmetry breaking. Also note the inevitable existence of a right-handed neutrino, which gives rise to a see-saw mechanism [5].

To achieve the desired symmetry breaking (see section 3) a  $(10, 1)$  representation of the Higgs field is used. An element of the Higgs field is denoted by the corresponding fermion assignment— $\bar{\nu}_H$  and so on.

The Lagrangian for this model is written as

$$\mathcal{L} = \mathcal{L}_f + \mathcal{L}_h + \mathcal{L}_g, \quad (6)$$

such that

$$\begin{aligned} \mathcal{L}_f &= i \sum_n (\bar{\mathbf{f}}_L^n \not{D} \mathbf{f}_L^n + \bar{\mathbf{f}}_R^n \not{D} \mathbf{f}_R^n), \\ \mathcal{L}_h &= \text{tr}[(D_\mu \Phi)^\dagger D^\mu \Phi] - V(\Phi), \\ \mathcal{L}_g &= \frac{1}{4} \text{tr}(F_{\mu\nu}^a F_a^{\mu\nu}) + \frac{1}{4} \text{tr}(\widetilde{F_{\mu\nu}} \widetilde{F^{\mu\nu}}), \end{aligned} \quad (7)$$

where the summation in  $\mathcal{L}_f$  is over the different fermion representations. The Higgs potential,  $V(\Phi)$ , is

$$V(\Phi) = \lambda_1 (\text{tr}(\bar{\Phi} \Phi) - \eta^2)^2 + \lambda_2 \text{tr}(\bar{\Phi} \Phi \bar{\Phi} \Phi). \quad (8)$$

Also, the covariant derivative is written as

$$D^\mu M_n = \partial^\mu M_n + g d_n (A_a^\mu T_a) M_n + \tilde{g} d_n (\widetilde{A^\mu T}) q_n M_n. \quad (9)$$

Note that in the above Lagrangian there are no Yukawa coupling terms for fermions to Higgs—this is because there are no gauge invariant Yukawa coupling terms.

### 3 Symmetry Breaking in Flipped $SU(5)$ .

For  $\eta^2 > 0$  and  $\lambda_1, (2\lambda_1 + \lambda_2) > 0$ , the Higgs potential (8) has a set of degenerate minima of the Higgs field corresponding to the vacuum manifold. Furthermore, for such ranges of the parameters the gauge group is transitive over the vacuum manifold. Thus, the vacuum manifold (denoted by  $\mathcal{M}_0$ ) can be written as

$$\mathcal{M}_0 = \{\Phi_c : V(\Phi_c) \text{ is a minimum.}\} \cong \frac{SU(5) \times \widetilde{U(1)}}{SU(3) \times SU(2) \times U(1)}. \quad (10)$$

The process of symmetry breaking through a phase transition is described by the familiar picture of the Kibble mechanism [6]. The Higgs field tries to relax to the vacuum to minimise its potential energy via taking a vacuum expectation value (VEV),  $\Phi_c \in \mathcal{M}_0$ . Due to the finite time over which the phase-transition takes place  $\Phi_c$  will not be uniform in space. Certain boundary conditions arise naturally and the Higgs field will relax to a state of minimum potential and kinetic energy. This leads to background configurations, which may or may not be stable. We study the configurations that occur in flipped  $SU(5)$  in the next section.

Since the gauge group is transitive over the vacuum manifold, a gauge rotation may be performed on the VEV so that, without loss of generality, the Higgs VEV can be in the  $\overline{\nu}_H$  direction,

$$\Phi_c = v \begin{pmatrix} 0 & 0 & 0 & \vdots & 0 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \vdots & 0 & 1 \\ 0 & 0 & 0 & \vdots & -1 & 0 \end{pmatrix}. \quad (11)$$

The size is found by minimising the Higgs potential, giving

$$v^2 = \frac{\lambda_1 \eta^2}{2\lambda_1 + \lambda_2}. \quad (12)$$

Locally the VEV can always be rotated to such a standard value. Although, globally such a situation does not always exist. This is the origin of background configurations.

A short calculation using eq. (7) yields the mass terms for gauge bosons to be

$$\mathcal{L}_{\text{gauge mass}} = \text{tr}[(d_{10}(gA_a^\mu T_a + g'\widetilde{A}^\mu \widetilde{T})\Phi_c)^\dagger (d_{10}(gA_a^\mu T_a + g'\widetilde{A}^\mu \widetilde{T})\Phi_c)]. \quad (13)$$

Now,  $d_{10}(T_a)H_c = 0$  for  $a = 1..8$  and  $a = 22, 23, 24$ . All other generators (including  $\tilde{T}$ ) create mass terms for the gauge fields. Thus, one identifies colour- $SU(3)$  as being generated by  $T_a : a = 1..8$  and isospin- $SU(2)$  as being generated by  $T_a : a = 22, 23, 24$ . Furthermore, analogously to the electroweak model, there is a linear combination of generators that gives rise to another massless gauge field, such that its generator lies perpendicular to the vacuum. This generator is a linear combination of  $\tilde{T}$  and  $T_{15}$ .

It should be noted that, since  $T_{15}$  is symmetric and  $\Phi_c$  is antisymmetric, then  $d_{10}(T_{15})\Phi_c = 2T_{15}\Phi_c$ . Thus, the minimal coupling of these generators to the Higgs vacuum is

$$d_{10}(A^\mu)\Phi_c = 2gA_{15}^\mu T_{15}\Phi_c + \tilde{g}\tilde{A}^\mu\tilde{T}\Phi_c, \quad (14)$$

with  $A^\mu = A_{15}^\mu T_{15} + \tilde{A}^\mu\tilde{T}$ .

Analogy with the electroweak model yields the hypercharge generator,  $T_Y$ , and a massive generator,  $T_V$  (the analogue of the Z-boson generator), by an orthogonal rotation of  $T_{15}$  and  $\tilde{T}$ . To give the same minimal coupling, with  $A^\mu = A_a^\mu T_a + Y^\mu T_Y + V^\mu T_V$ , the corresponding rotation of the gauge fields is

$$\begin{pmatrix} A_{15}^\mu \\ \tilde{A}^\mu \end{pmatrix} = \begin{pmatrix} \cos \Theta & -\sin \Theta \\ \sin \Theta & \cos \Theta \end{pmatrix} \begin{pmatrix} Y^\mu \\ V^\mu \end{pmatrix}, \quad (15)$$

$$\begin{pmatrix} gT_{15} \\ \tilde{g}\tilde{T} \end{pmatrix} = \begin{pmatrix} \cos \Theta & -\sin \Theta \\ \sin \Theta & \cos \Theta \end{pmatrix} \begin{pmatrix} g_Y T_Y \\ g_V T_V \end{pmatrix}.$$

For hypercharge to be a gauge symmetry of the Standard Model,  $T_Y$  must be perpendicular to the vacuum,  $d_{10}(T_Y)\Phi_c = 0$ . This is so only for

$$\tan \Theta = \frac{\tilde{g}}{g}. \quad (16)$$

With this angle, squaring eq. (15) yields the couplings to the hypercharge and V-bosons to be

$$g_Y = \sqrt{\frac{g^2 \tilde{g}^2}{g^2 + \tilde{g}^2}}, \quad g_V = \sqrt{\frac{g^4 + \tilde{g}^4}{g^2 + \tilde{g}^2}}. \quad (17)$$

The gauge couplings to the Lepto-quark gauge bosons and the gauge fields of  $SU(2)$  and  $SU(3)$  are  $g$ . One interprets  $\Theta$  as a generalised Weinberg angle appertaining to GUT's.

Thus the hypercharge generator and the V-generator are defined from  $T_{15}$  and  $\tilde{T}$  to be

$$\begin{aligned} T_Y &= \frac{1}{\sqrt{2}}(\tilde{T} + T_{15}) \\ T_V &= \frac{\tilde{g}^2}{\sqrt{g^4 + \tilde{g}^4}}\tilde{T} - \frac{g^2}{\sqrt{g^4 + \tilde{g}^4}}T_{15} \end{aligned} \quad (18)$$

A convenient check on the above calculation is when  $g = \tilde{g}$ , then necessarily  $g_Y = g_V = g$ .

To check the fermion assignments of Section 2 (eqs. (3), (4) and (5)), the hypercharges have to be verified to be correct. The fermion hypercharges are given by the eigenvalues of the operator  $d_n(T_Y)f_L^n$ . Using eq. (17) plus a little algebra this becomes

$$id_n(T_Y)f_L^n = \frac{g \cos \Theta}{g_Y}[d_n(T_{15}) + \sqrt{\frac{12}{5}}(iq_n \frac{\tilde{g}}{g} \tan \Theta)]f_L^n. \quad (19)$$

Using this the hypercharges of the fermions are

$$\begin{aligned} id_1(T_Y)f_L^1 &= -\sqrt{15}2f_L^1, \\ id_{\bar{5}}(T_Y)\bar{f}_L^{\bar{5}} &= -\sqrt{15}\text{diag}(-4/3, -4/3, -4/3; -1, -1)\bar{f}_L^{\bar{5}}, \\ id_{10}(T_Y)f_L^{10} &= -\sqrt{15}\text{diag}(2/3; 1/3; 0)f_L^{10}; \end{aligned} \quad (20)$$

which verifies that the correct particle assignments have been made. There is a normalisation factor  $\sqrt{15}$  which is included in the definition of the coupling constant.

For completeness, we give details of the Higgs mechanism and the masses of the bosonic sector of the theory in appendix B.

## 4 Embedded Defects in Flipped $SU(5)$ .

As we have seen from the last section, the flipped  $SU(5)$  model is very similar to the electroweak model in the pattern of symmetry breaking. Both theories are of the form  $SU(n) \times U(1)$  and break to a group which has a  $U(1)$  factor between the  $SU(n)$  and the  $U(1)$  parts. In the electroweak model this structure gives a non-trivial embedded defect structure; yielding W-strings [9] and Z-strings [7], which are stable in the semi-local limit [10]. Thus, it seems sensible to suppose these structures also exist in flipped  $SU(5)$ . Furthermore, it is known that in the electroweak



model Z-strings are unstable for physical Weinberg angle [11]. However, for GUT's the parameters are different and the embedded defect might well be stable. This situation is particularly favourable in flipped  $SU(5)$  owing to the coupling constants not being required to meet at unification. This situation is analysed in the next section.

We shall show that for flipped  $SU(5)$  there are two classes of gauge inequivalent embedded Nielsen-Olesen solutions. One class contains one element (the analogue of the Z-string); the other class contains twelve elements (the analogues of the W-strings). We refer to the one-dimensional class of embedded vortices as ‘V-strings’ — because these are generated by the generator that gives the V-boson.

The stability of such embedded defects is a very important issue and we shall show the analogy between the Z-string of electroweak theory and the V-string may be carried further; namely that the V-string is stable in an appropriate semi-local limit.

## 4.1 Existence of Embedded Defects in Flipped $SU(5)$ .

We follow the approach of [12] for describing the existence of embedded defect solutions

To describe the embedded solutions we need a reference point on the vacuum manifold in order to generate the solution and to describe different, but possibly gauge equivalent, solutions. We take this point to be, without loss of generality,  $\Phi_c$  as given by eq. (11). For convenience we also need a basis for  $L(SU(5) \times \widetilde{U(1)})$  orthonormal with respect to the natural Inner Product; we take this basis to be as given in the appendix.

To define the embedded solutions one considers one-parameter subgroups  $G^{\text{emb}} \subseteq SU(5) \times \widetilde{U(1)}$ , such that its corresponding little group  $H^{\text{emb}} = G^{\text{emb}} \cap \{SU(3)_C \times SU(2)_I \times U(1)_Y\}$  is trivial. These one-parameter groups are generated by broken generators, i.e. those generators which satisfy

$$d_{10}(T_a)\Phi_c \neq 0. \quad (21)$$

We denote this vector space of broken generators by  $M$  and it is spanned by the Lepto-Quark generators and by the V-boson generator. In the basis previously

mentioned

$$M = \text{span}\{T_V, T_a : a = 9..14, 16..21\}. \quad (22)$$

Then for each  $T_s \in M$  one has a one-parameter subgroup of  $SU(5) \times \widetilde{U(1)}$ , given by

$$G^{\text{emb}}[T_s] = \{g(\theta) = \exp\left(\frac{\theta T_s}{c(T_s)}\right) : \theta \in [0, 2\pi)\}, \quad (23)$$

where  $c(T_s)$  is a normalisation constant such that  $g(2\pi) = \text{id}_G$  and there does not exist a  $\theta_0 \in (0, 2\pi)$  with  $g(\theta_0) = \text{id}_G$  (we are here denoting the identity of the gauge group by  $\text{id}_G$ ).

We then define an embedded subspace of the representation-space,  $\mathcal{V}^{\text{emb}}[T_s] \subseteq \mathcal{V}$  by

$$\mathcal{V}^{\text{emb}}[T_s] = \{\alpha \exp\left(\frac{\theta d_{10}(T_s)}{c(T_s)}\right) \Phi_c : \theta \in [0, 2\pi), \alpha \in \mathcal{R}\text{e}\}. \quad (24)$$

It is then clear that, under this construction,  $\mathcal{V}^{\text{emb}}[T_s]$  is invariant under the action of  $G^{\text{emb}}[T_s]$ . Hence, we have constructed a unique embedded sub-theory from the element  $T_s \in M$ , that is defined by  $(G^{\text{emb}}[T_s], \mathcal{V}^{\text{emb}}[T_s])$ . This embedded sub-theory has the property that its little group  $H^{\text{emb}} = G^{\text{emb}} \cap \{SU(3)_C \times SU(2)_I \times U(1)_Y\}$  is trivial.

The vacuum manifold for the embedded sub-theory is defined to be

$$\mathcal{M}_0^{\text{emb}}[T_s] = \mathcal{M}_0 \cap \mathcal{V}^{\text{emb}}[T_s] \quad (25)$$

and one sees that all such embedded vacuum manifolds have non-trivial first homotopy groups, since

$$\mathcal{M}_0^{\text{emb}}[T_s] \cong \frac{G^{\text{emb}}[T_s]}{H^{\text{emb}}[T_s]} \cong S^{(1)}. \quad (26)$$

If one considers just the embedded sub-theory in isolation, then it exhibits Nielsen-Olesen solutions due to its non-trivial topological nature. These embedded solutions are given by (in the temporal gauge):

$$\begin{aligned} \Phi(r, \theta) &= f_{\text{NO}}(r) D_{10}(\exp(\frac{T_s \theta}{c(T_s)})) \Phi_c, \\ \underline{A} &= \frac{g_{\text{NO}}(r)}{g_s r} \left(\frac{T_s}{c(T_s)}\right) \hat{\underline{\theta}}, \\ A^0 &= 0. \end{aligned} \quad (27)$$

Here  $f_{\text{NO}}(r)$  and  $g_{\text{NO}}(r)$  are the profile functions for a Nielsen-Olesen solution and satisfy

$$\begin{aligned} f_{\text{NO}}'' + \frac{f_{\text{NO}}'}{r} - \frac{(1 + g_{\text{NO}})^2}{r^2} f_{\text{NO}} &= -f_{\text{NO}}(m_1^2 + 2(\lambda_1 + \lambda_2/2)f_{\text{NO}}^2), \\ \left(\frac{-1}{c(T_s)^2}\right)(g_{\text{NO}}'' - \frac{g_{\text{NO}}'}{r}) &= -4g_S^2 v^2 (g_{\text{NO}} + 1)f_{\text{NO}}^2, \end{aligned} \quad (28)$$

where this equation has been derived from the equations of motion.

When this embedded solution is taken back to the full theory it still remains a solution provided that

$$(\Phi(r, \theta), \mathcal{V}^\perp) = 0, \quad (29)$$

where  $\mathcal{V}^\perp = \{\Phi \in \mathcal{V} : (\Phi, \mathcal{V}^{\text{emb}}) = 0\}$  and

$$(d_{10}(A^\mu), d_{10}(T_s^\perp)) = 0, \quad (30)$$

with  $T_s^\perp = \{T \in L(SU(5) \times \widetilde{U(1)}) : (T, T_s) = 0\}$ .

It can be shown [12] that these conditions are generally satisfied if

$$d_{10}(T_s)\Phi \in \mathcal{V}^\perp, \text{ for } \Phi \in \mathcal{V} \text{ and } T \in T_s^\perp, \quad (31)$$

which we can verify is satisfied for flipped  $SU(5)$ .

Before writing down the embedded solution we should firstly derive the condition for two different embedded sub-theories (defined by, say,  $T_s$  and  $T'_s$ ) to be gauge equivalent. We shall deal with gauge transformations of the type

$$\begin{aligned} \Phi &\mapsto D_{10}(g)\Phi, \\ A^\mu &\mapsto \text{Ad}(g)A^\mu, \\ \text{such that } g &\in SU(3)_C \times SU(2)_I \times U(1)_Y. \end{aligned} \quad (32)$$

A short calculation shows that this is equivalent to a rotation of the generator,  $T_s$ , which defines the sub-theory, of the form

$$T_s \mapsto \text{Ad}(g)T_s. \quad (33)$$

In other words

$$T_s \mapsto e^T T_s e^{-T}, \text{ with } T \in L(SU(3)_C) \oplus L(SU(2)_I) \oplus L(U(1)_Y). \quad (34)$$

Therefore, two embedded solutions (with string generators  $T_s$  and  $T'_s$ , say) are gauge equivalent if there exists a  $g \in SU(3)_C \times SU(2)_I \times U(1)_Y$  such that  $T'_s = D_{10}(g)T_s$ .

For flipped  $SU(5)$  it is simple, but tedious, to verify that the generators of the sub-theory,  $T \in M$ , split into two gauge inequivalent classes under eq. (33). Then

$$\begin{aligned} M &= M_1 \oplus M_2, \\ \text{with } M_1 &= \text{span}\{T_V\}, \\ \text{and } M_2 &= \text{span}\{T_a : a = 9..14, 16..21\}, \end{aligned} \tag{35}$$

and for  $T, T' \in M_2$  there exists a  $g \in SU(3)_C \times SU(2)_I \times U(1)_Y$  such that  $T = D_{10}(g)T'$ .

Hence, for flipped  $SU(5)$  there are two gauge inequivalent classes of solution. Firstly, the V-string solution

$$\begin{aligned} \Phi(r, \theta) &= f_{\text{NO}}(r) D_{10}(\exp\left(\frac{T_V \theta}{c(T_V)}\right)) \Phi_c = f_{\text{NO}}(r) e^{i\theta} \Phi_c, \\ \underline{A} &= \frac{g_{\text{NO}}(r)}{g_V r} \left(\frac{T_V}{c(T_V)}\right) \hat{\underline{\theta}}, \\ A^0 &= 0 \end{aligned} \tag{36}$$

with  $c(T_V)$  a calculable constant dependent upon gauge couplings. The other class of solutions are the Lepto-Quark strings and are generated by  $T_{\text{LQ}} \in M_2$ , having the solution

$$\begin{aligned} \Phi(r, \theta) &= f_{\text{NO}}(r) D_{10}(\exp\left(\frac{T_{\text{LQ}} \theta}{c(T_{\text{LQ}})}\right)) \Phi_c, \\ \underline{A} &= \frac{g_{\text{NO}}(r)}{g_{\text{LQ}} r} \left(\frac{T_{\text{LQ}}}{c(T_{\text{LQ}})}\right) \hat{\underline{\theta}}, \\ A^0 &= 0. \end{aligned} \tag{37}$$

and as shown previously all Lepto-Quark strings are gauge equivalent.

## 4.2 The Semi-Local Limit of the V-string.

That there was a resurgence of interest in the Z-string of electroweak theory was due to the fact that it is stable in the semi-local limit of the Weinberg-Salam model [10], but not for physical Weinberg angle [11]. Taking the analogy between flipped  $SU(5)$  and electroweak theory, one would expect that the V-string should be stable

in the semi-local limit. Furthermore, the domain of stability *might* overlap the point of physical reality in a GUT scale scenario — this is discussed in the next section.

Work by Preskill in [13] has shown how to identify stable semi-local defects when one considers a general symmetry breaking  $G \rightarrow H$ . Consider a subgroup  $G_{\text{gauge}} \subseteq G$  and  $H_{\text{gauge}} = G_{\text{gauge}} \cap H$ . If one considers  $G$  to be a global symmetry (zero gauge-coupling) and then gauges just  $G_{\text{gauge}}$ , the existence of semi-local defects is indicated by the non-trivial homotopy classes of  $G_{\text{gauge}}/H_{\text{gauge}}$ . Semi-local vortices, which have a corresponding non-trivial first homotopy class, are stable.

Hence for flipped  $SU(5)$  the appropriate semi-local limit of the model is when  $\Theta_{\text{GUT}} \rightarrow \frac{\pi}{2}$ , so that just the  $\widetilde{U(1)}$  symmetry is gauged. Since  $\widetilde{U(1)} \cap H = \text{id}_G$ , the topology of the situation gives stable semi-local defects generated by  $\tilde{T}$ . It is clear that these semi-local defects correspond (with a gauge transformation) to the V-strings previously discussed.

## 5 Discussion of Results.

Owing to extremely accurate measurements of the Z mass, the electroweak Weinberg angle, electric charge and strong charge, the low energy coupling constants for  $SU(3)_c$ ,  $SU(2)_I$  and  $U(1)_Y$  are known to an unprecedented degree of accuracy. Using renormalisation group techniques the high energy values of these couplings can be calculated. GUT's, in general, have some constraints on the form of couplings and also have a lower bound upon the unification energy scale (from proton decay rates). We are thus in a position to apply some physics to the existence of stable V-strings. The results on running couplings are taken from [3].

### 5.1 Coupling Constant Unification.

For GUT's that have simple gauge groups, coupling constants are required to meet at unification. However, GUT's with a non-simple gauge group are only constrained such that the strong and weak couplings must meet. Thus from calculations of running couplings one may determine the unification scale. Furthermore, from the value of the hypercharge coupling constant at unification one may calculate the value of  $\tan\Theta_{\text{GUT}}$ , which is the quantity relevant for stability of V-strings.

Denoting the hypercharge coupling by  $\alpha_1$ , the weak (isospin) coupling by  $\alpha_2$  and the strong coupling by  $\alpha_3$ , renormalisation group calculations yield the graph for running couplings in [3].

The unification scale is determined from the energy when  $\alpha_2 = \alpha_3$ , thus for flipped  $SU(5)$  it is identified to be

$$\mu_{\text{GUT}} = 10^{16} \text{ to } 10^{17} \text{ GeV.} \quad (38)$$

and at this scale the coupling constants are

$$\alpha_{2\text{GUT}}^{-1} = \alpha_{3\text{GUT}}^{-1} = 45 \text{ to } 49, \quad \alpha_{1\text{GUT}}^{-1} = 36 \text{ to } 38. \quad (39)$$

To calculate the value of  $\tan\Theta_{\text{GUT}}$  one needs the value of the  $SU(5)$  coupling constant,  $g$  and the  $\widetilde{U(1)}$  coupling constant  $\tilde{g}$ . These can be easily calculated from  $\alpha_{i\text{GUT}}$ , then a simple calculation yields

$$\tan\Theta_{\text{GUT}} = \frac{\tilde{g}}{g} = 1.2 \text{ to } 1.4. \quad (40)$$

The main uncertainty is in the value of the strong coupling constant. However, the above quantities are enough for some simple calculations.

The non-supersymmetric version of flipped  $SU(5)$ , where the coupling constants do not meet, necessarily precludes the existence of a further unification to one coupling constant (i.e. a simple group) at a higher energy-scale. This is because  $SU(5)$  is asymptotically free and the  $\widetilde{U(1)}$  is not. Thus after unification the coupling constant diverge, precluding further unification into a simple group. This is a generic feature of any unification scheme of a similar form to non-supersymmetric flipped  $SU(5)$ .

## 5.2 Stability of V-strings.

We have shown that the V-string has a stable semi-local limit. The stability of the corresponding semi-local vortex is dependant upon the value  $\lambda = (\lambda_1 + \lambda_2/2)$ , which parameterises the strength of the potential. For  $\lambda \in (0, 1)$  the semi-local vortex is stable and for  $\lambda \geq 1$  the vortex is unstable.

There is another line of interest in the  $(\lambda, \Theta_{\text{GUT}})$ -stability plane, which is for  $(\lambda, \Theta_{\text{GUT}} = \pi/4)$ , where the embedded vortex is unstable [12].

The region of stability is separated from the region of instability by a critical curve going from  $(\lambda = 1, \Theta_{\text{GUT}} = \pi/2)$  to  $(\lambda = 0, \Theta_{\text{GUT}} = \Theta_{\text{crit}})$ , with  $\Theta_{\text{crit}} \in [\pi/4, \pi/2)$ . To see an example of such a stability region, one should refer to [11], which calculates the form of the stability region for electroweak Z-strings of the Weinberg-Salam model. Note that  $\Theta_{\text{crit}}$  is not obtained from the numerical techniques used to evaluate these graphs. It is an open question what the value of  $\Theta_{\text{crit}}$  is. Another stability analysis, for the case of the two-doublet Electroweak model, has been performed in [14].

From comparing the results in [14] and [11], it is clear that as the number of Higgs degrees of freedom increase, then the region of stability gets larger. The physical reason for this is related to observing that stability depends upon whether the potential decreases more than the kinetic terms upon small perturbations in the Higgs field. This is why embedded defects are unstable for large  $\lambda$ . Having more Higgs spreads the perturbation over more degrees of freedom — having no effects on changes in the potential, but causing the kinetic terms to increase more. Thus more Higgs degrees of freedom increases the region of stability.

Hence, for flipped  $SU(5)$  it seems likely (or at least an open question) that it has stable embedded defects. Firstly, because flipped  $SU(5)$ , being a GUT, has many Higgs degrees of freedom. Secondly, because  $\Theta_{\text{GUT}}$  is large (certainly larger than  $\pi/4$ ).

### 5.3 The Cosmology of Flipped $SU(5)$ .

First of all, we point out that the unification energy scale is compatible with present proton-decay rates. Thus flipped  $SU(5)$  remains a viable option.

It is a well known feature of flipped  $SU(5)$  that, provided it is not embedded in a simple gauge group, there are no topological monopoles. There are, however, embedded monopole solutions — but these are unstable. Thus, the monopole problem is circumvented in flipped  $SU(5)$ .

The presence of stable embedded defects gives many cosmological implications. We firstly describe the effect of cooling upon these defects before considering general cosmological implications.

The first point to note for the cosmology is that the probability of formation

(per unit volume) is less for an embedded defect than for an Abelian topologically stable defect. For an Abelian defect the Kibble-mechanism [6] says that we only need to consider random phases in the  $U(1)$  factor of the gauge group. However, for an embedded defect the Kibble mechanism would predict that some combination of a Lepto-quark string and a V-string would be initially formed. This configuration is not a stationary point of the Lagrangian and so some currents must be present to compensate for this. This combination would form probabilistically either a Lepto-quark string or a V-string. Thus the probability of formation of a V-string must be less than that for an Abelian string. The probability of formation is also decreased by the region of stability [15]

As the universe cools the coupling constants run and  $\tan\Theta_{GUT}$  decreases. Hence, it appears that a stable embedded defect would destabilise at a lower energy scale. This does not appear to be the case. Observe that in the centre of the defect symmetry is restored and hence it is this  $\tan\Theta_{GUT}$  that is relevant to stability and not the low temperature value. This should stop a stable embedded defect from decaying due to the gauge couplings running as the temperature falls.

However, embedded defects are not topologically stable; they can only be dynamically stable. This means there is no topological charge to guarantee the lasting stability of such a defect and in general such a defect will decay by the nucleation of an embedded monopole and anti-monopole pair along the string [16]. A string ending in a monopole and anti-monopole will then shrink due to tension along it. It is an open question whether a short length of dynamically stable string ending at such monopole pairs would be stable or not. If such a configuration were completely stable then it might over-close the Universe — resulting in a cosmological disaster!

The rate of creation of monopole and anti-monopole pairs should be fairly high on a length of V-string. As the parameters of an embedded defect get closer to the region of dynamical instability then the probability of nucleation of monopole and anti-monopole pairs gets higher. Thus, it is unlikely that a length of V-string would survive until today. However, the rate of nucleation may be low enough to ensure that stable configurations consisting of short lengths of V-string ending in monopoles may be in quite small abundance. This abundance could be small enough to circumvent the over-closure problem.



Gravitational interactions of a stable embedded defect are the same as topologically stable defects. Thus, if V-strings were to survive into the matter-dominated epoch of the Universe then they would seed structure formation. Also, a V-string would produce gravitational lensing effects upon light travelling past it. Due to the (reasonably) short lifetime of a V-string it is unlikely that it would survive long enough to produce these effects. However, they may survive long enough to pass through the surface of last scattering — leaving a signature upon the Cosmic microwave background. This last effect is probably the only way of observing such strings directly.

If V-strings were able to live to the Electroweak phase transition they should give Electroweak baryogenesis. It has been recently shown that topological strings, in passing through the Electroweak phase-transition, give baryogenesis with the bias given by  $CP$  [17]. A V-string has the same interaction with matter as an Abelian topological string and thus they would produce a similar effect.

It is clear from the above that the V-string, if it were stable, could be Cosmologically very interesting. The crucial question is, however, how long do they live? Too short a lifetime and their Cosmological consequences could be minimal (or fatal if embedded monopole and anti-monopole pairs joined by short V-strings were stable). A long lifetime and then we have a realistic GUT producing strings with lots of nice Cosmological effects.

We should point out that if stable V-strings were to be observed (or indeed stable GUT-scale embedded defects of any form) then this would contradict supersymmetry. Supersymmetry causes the coupling constants to meet at unification — meaning that any GUT-scale embedded defect will be unstable.

It is worth noting that if a Cosmological consequence turns out to be particularly dire (such that it rules out our Universe) then the particular model that gives rise to this consequence is ruled out (or revamped with some extra/different parameters). Thus the consequences that we have sketched could be fatal for flipped  $SU(5)$ . So with all the glories and nice features of flipped  $SU(5)$ , it is possible that embedded defects could rule it out.

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# Appendix A.

The modified Gell-Mann basis for  $L(SU(5))$  is [18]

$$T_a = \frac{i}{\sqrt{2}}\mu_a.$$

The Inner Product on this vector space is given by  $(T_a, T_b) = \text{tr}(T_a^\dagger T_b)$ . In order for  $T_a$  to be orthonormal with respect to this basis,  $\mu_a$  with  $a = 1..24$  is defined to be:

$$\begin{aligned} \mu_1 &= \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, & \mu_2 &= \begin{pmatrix} 0 & -i & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, & \mu_3 &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \\ \mu_4 &= \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, & \mu_5 &= \begin{pmatrix} 0 & 0 & -i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, & \mu_6 &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \\ \mu_7 &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & 0 \\ 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, & \mu_8 &= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, & \mu_9 &= \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \\ \mu_{10} &= \begin{pmatrix} 0 & 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, & \mu_{11} &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, & \mu_{12} &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \\ \mu_{13} &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, & \mu_{14} &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 \\ 0 & 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, & \mu_{15} &= \sqrt{\frac{3}{5}} \begin{pmatrix} 2/3 & 0 & 0 & 0 & 0 \\ 0 & 2/3 & 0 & 0 & 0 \\ 0 & 0 & 2/3 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}, \\ \mu_{16} &= \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{pmatrix}, & \mu_{17} &= \begin{pmatrix} 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 \end{pmatrix}, & \mu_{18} &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}, \end{aligned}$$

$$\begin{aligned}
\mu_{19} &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 & 0 \end{pmatrix}, & \mu_{20} &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}, & \mu_{21} &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & i & 0 & 0 \end{pmatrix}, \\
\mu_{22} &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}, & \mu_{23} &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & i & 0 \end{pmatrix}, & \mu_{24} &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}.
\end{aligned}$$

## Appendix B.

The Higgs fields gain mass via their coupling to the vacuum manifold. They do this by ‘eating’ components of the Higgs field—this can be seen by transforming to the unitary gauge where it is transparent that longitudinal polarisation component of the massive gauge fields are from the Higgs degrees of freedom. The eaten components of the Higgs field correspond to fields that are transverse to the vacuum manifold. The rest of the Higgs field—which correspond to massive Goldstone bosons—is from the radial Higgs fields  $\Phi_R$ , such that

$$\begin{aligned}
\Phi_R &= \{\Phi_R = (\Phi - \Phi_c) : \text{tr}(\Phi_R^\dagger d_{10}(T)\Phi_c) = 0, \\
&\quad T = T_a, T_V, T_V, \text{ for } a = 1..23\}.
\end{aligned} \tag{41}$$

This corresponds to a choice of generalised polar coordinates in specifying the Higgs field. A short calculation yields

$$\Phi_R = \begin{pmatrix} 0 & \bar{d}_H^1 & -\bar{d}_H^2 & \vdots & 0 & 0 \\ -\bar{d}_H^1 & 0 & \bar{d}_H^3 & \vdots & 0 & 0 \\ \bar{d}_H^2 & -\bar{d}_H^3 & 0 & \vdots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \vdots & 0 & \sigma_H \\ 0 & 0 & 0 & \vdots & \sigma_H & 0 \end{pmatrix}, \tag{42}$$

with  $\bar{d}_H = \sigma_H e^{i\theta}$  and  $\sigma_h \in \mathcal{R}$ . The other components of the Higgs field are represented by a gauge rotation of this, yielding

$$\Phi = \Phi_c + d_{10}(g)\Phi_R, \tag{43}$$

where  $g \in SU(5) \times \widetilde{U(1)}$ .

To obtain the mass terms for gauge bosons and Higgs one substitutes  $\Phi$  in  $\mathcal{L}_h$  (7). The unitary gauge is implicitly used to transform away transverse components of the Higgs field, obtaining

$$\begin{aligned}\mathcal{L}_h &= \mathcal{L}_{\text{gauge mass}} + \mathcal{L}_{\text{Higgs mass}} + \mathcal{L}_I, \\ \mathcal{L}_{\text{gauge mass}} &= g^2 v^2 \sum_{i=1}^3 (\overline{X}_i^\mu X_{i\mu} + \overline{Y}_i^\mu Y_{i\mu}) + \frac{32}{35} g_Y^2 v^2 \overline{V}^\mu V_\mu, \\ \mathcal{L}_{\text{Higgs mass}} &= -(m_1^2 + 8v^2) \left( \frac{\lambda_1}{2} + \lambda_2 \right) \left( \sum_{i=1}^3 \overline{u}_{iH} u_{iH} + \overline{\sigma}_H \sigma_H \right),\end{aligned}\tag{44}$$

with  $\mathcal{L}_I$  being the interaction between gauge and Higgs fields. The Lepto-quark bosons are  $X^\mu, Y^\mu$ , as described below. Hence the masses of the respective particles are

$$m_X^2 = m_Y^2 = g^2 v^2, \quad m_V^2 = \frac{32}{35} g_Y^2 v^2, \quad m_H^2 = -(m_1^2 + 8v^2) \left( \frac{\lambda_1}{2} + \lambda_2 \right).\tag{45}$$

Note that since  $\lambda_1, \lambda_2 \sim 1$  and  $-m_1^2 \sim v^2$ , the coefficient of the Higgs mass term is positive.

For completeness we shall describe which Higgs fields are eaten by which gauge bosons. To make this explicit, choose a basis where the Lepto-Quark gauge bosons have the direction  $T_{iX} = \frac{1}{\sqrt{2}}(T_{2i+7} + iT_{2i+8})$ ,  $T_{iY} = \frac{1}{\sqrt{2}}(T_{2i+14} + iT_{2i+15})$  in gauge space. The gauge fields are similarly related to the  $A_i^\mu$ . Then

$$A^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} & & & \vdots & \overline{X}_1^\mu & \overline{Y}_1^\mu \\ SU(3) \text{ gauge fields} & & & \vdots & \overline{X}_2^\mu & \overline{Y}_2^\mu \\ & & & \vdots & \overline{X}_3^\mu & \overline{Y}_3^\mu \\ \dots & \dots & \dots & \dots & \dots & \dots \\ X_1^\mu & X_2^\mu & X_3^\mu & \vdots & SU(2) \text{ gauge} & \\ Y_1^\mu & Y_2^\mu & Y_3^\mu & \vdots & \text{fields} & \end{pmatrix} + Y^\mu T_Y + V^\mu T_V.\tag{46}$$

From the minimal coupling (14) it is clear that  $\frac{1}{\sqrt{2}}(X_i^\mu + i\overline{X}_i^\mu)$  eats the real part of  $d_H^i$  and its conjugate eats the imaginary part. Similarly, the Y-bosons eat  $u_H^i$ . The V-bosons eat the  $\sigma_H$ -field.

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